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RM-6031

B. E. - I (Sem. II) (All) Examination

May / June – 2010

Engineering Mathematics - II

Time : **3** Hours] [Total Marks : **100**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **B. E. - 1 (Sem. 2) (All)**

Name of the Subject :
☛ **Engineering Mathematics - 2**

☛ Subject Code No. : **6 0 3 1** ☛ Section No. (1, 2,.....) : **1&2**

Seat No. :

Student's Signature

(2) All questions are **compulsory**.

(3) Write each section in **separate** answer books.

SECTION - I

1 (a) Do as directed : **10**

(1) If $u = \log x + \log y$ then $xu_x + yu_y =$ ________.

(2) Find the equation of tangent plane to the surface $2x^2 + y^2 + 2z = 3$ at $(2, 1, -3)$.

(3) If $u = 2xy$, $v = x^2 - y^2$, $x = r\cos\theta$, $y = r\sin\theta$ find $\dfrac{\partial(u, v)}{\partial(r, \theta)}$.

(4) Write the iteration formula for Newton-Raphson method.

(5) To use both Simpson's 1/3 and 3/8 rules, the given interval should be divided into how many numbers of subintervals ?

(b) Attempt the following :

(1) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, show that $xu_x + yu_y = \sin 2u$ **6**

and $x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = 2 \cos 3u \sin u$.

(2) Use Gauss-Seidel method to solve : **4**

$2x + y + z = 4$

$x + 2y + z = 4$

$x + y + 2z = 4$

2 Attempt any **four** : **16**

(1) If $u = f(x + at) + g(x - at)$ prove that $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$

(2) If $u = \sin^{-1}(x - y)$, $x = 3t$, $y = 4t^3$, show that $\frac{du}{dt} = \frac{3}{\sqrt{1 - t^2}}$.

(3) If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$ prove that

$\frac{1}{2}\frac{\partial u}{\partial x} + \frac{1}{3}\frac{\partial u}{\partial y} + \frac{1}{4}\frac{\partial u}{\partial z} = 0$

(4) Expand $\tan^{-1}\frac{y}{x}$ about the point $(1, 1)$

(5) Obtain the percentage error in the area of an ellipse when an error of 1% is made in measuring the semi major and semi minor axes.

3 (a) Attempt any **one** : **4**

(1) Find the extremum value of $x^3 + y^3 - 3axy, a > 0$.

(2) Show that the maximum value of $u = x^p y^q z^r$ when the variables x, y, z are subject to the condition

$ax + by + cz = p + q + r$ is $\left(\frac{p}{a}\right)^p + \left(\frac{q}{b}\right)^q + \left(\frac{r}{c}\right)^r$.

(b) Find the root of any **two** : **6**

(1) $x^2-4x-10=0$ correct to two decimal places by using Bisection method.

(2) $x^3-4x-9=0$ correct to two decimal places using method of false position.

(3) $\cos x = 3x-1$ correct to three decimal places using iteration method.

(4) $\sin x = 1-x$ correct to three decimal places using Newton-Raphson method.

(c) Attempt any **one** : **4**

(1) Evaluate $\int_0^1 x^3\,dx$ by using (i) Trapezoidal rule (ii) Simpson's 1/3 rule.

(2) Using Taylor's series method solve the IVP $\frac{dy}{dx}=3x+\frac{y}{2}$, $y(0)=1$ and find $y(0.1)$.

(3) Using Picard's method solve $\frac{dy}{dx}=1+xy$, $y(0)=0$ upto 3rd approximation.

SECTION - II

4 (a) Do as directed : **10**

(1) Find P.I. of $\left(D^2+5D+6\right)y=5$

(2) Define the Cauchy's linear differential equation with variable coefficients.

(3) Give the general solution obtained by the method by the variation of parameters.

(4) Define the following terms :

(a) Regular-singular point

(b) Ordinary point.

(5) Find the general solution of $y''-k^2y=0$; $(k\neq 0)\,y(0)=1,\ y'(0)=1$.

(b) Attempt the following :

(1) Prove that $\frac{1}{f(D)^2}\sin ax = \frac{1}{f(-a)^2}\sin ax$; $f\left(-a^2\right) \neq 0$ **4**

and discuss the case when $f(-a)^2 = 0$.

(2) An electric circuit consists of an inductance 0.1 H, a resistance of 20 ohm and a condenser of capacitance $25\,\mu$. Find the charge q and the current I at any time t, given that at $t = 0$, $q = 0.05$ coulomb and $i = \frac{dq}{dt} = 0$ when $t = 0$. **6**

5 (a) Attempt the following : **6**

(1) $y'' + 25y = 24\sin t,\ y(0) = 1,\ y'(0) = 1$

(2) $\left(D^2 + D + 1\right)y = e^t$

(3) $\left(D^2 + D\right)y = 1 + \cos t$

(b) Attempt any **two** of the following : **8**

(1) $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 3y = x^2\log x$

(2) $x^2y'' + xy' - 9y = 48x^5$

(3) Solve $y'' + 9y = \operatorname{cosec} 3x$ by M.V.P.

6 (a) Find the series solution of the following using Frobenius method : **10**

(1) $x^2y'' + xy' + \left(x^2 - 4\right)y = 0$

(2) $xy'' - 3y' + xy = 0$

(3) $x(x-1)y'' + (3x-1)y' + y = 0$

(b) Attempt any **one** of the following : **6**

(1) A beam of length L carries a transverse uniform load w per unit length. Find the equation of the deflection curve and maximum deflection when one end of the beam is clamped and the other is simply supported.

(2) Formulate a differential equation model for the LCR circuit with voltage source. Obtain its solution. Analyze the model and write the interpretations.